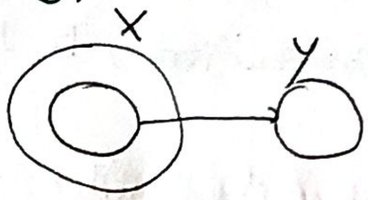
Proprietăți ale dependențelor funcționale

FD1 (Reflexivitate) Dc. $Y \subseteq X$, atunci r satisface $X \to Y$, $\forall r \in U$.



Dem. $X = (A_1, A_2, \dots, A_m)$

$Y = (A_{i1}, A_{i2}, A_{i3}, \dots, A_{ik})$ $\quad 1 \le i_1 \le i_2 \dots \le i_k \le m$

Fie $t_1, t_2 \in r$ a.î. $t_1[X] = t_2[X] \Rightarrow \left.\begin{array}{l} t_1[A_1] = t_2[A_1] \\ t_1[A_2] = t_2[A_2] \\ \vdots \\ t_1[A_m] = t_2[A_m] \end{array}\right\} \Rightarrow \left\{\begin{array}{l} t_1[A_{i1}] = t_2[A_{i1}] \\ \vdots \\ t_1[A_{ik}] = t_2[A_{ik}] \end{array}\right. \Rightarrow t_1[Y] = t_2[Y]$

Fie $t_1, t_2 \in r$ a.î. $t_1[X] \neq t_2[X] \Rightarrow$ în acest caz $X \to Y$, indiferent de mulț. Y

FD2 (Extensie) Dc. r satisface $X \to Y$ și $Z \subseteq W$, atunci r satisface $XW \to YZ$.

$$\frac{X \to Y,\ Z \subseteq W}{XW \to YZ}$$

$X = (A_1, A_2, \dots, A_m)$

$Y = (B_1, B_2, \dots, B_m)$

$W = (C_1, C_2, \dots, C_p)$

$Z = (C_{i1}, C_{i2}, \dots C_{ik})$ $\quad 1 \le i_1 \le i_2 \dots \le i_k \le p$

$X \to Y \Rightarrow \forall t_1, t_2 \in r$ cu $t_1[X] = t_2[X] \Rightarrow t_1[Y] = t_2[Y]$

$\Leftrightarrow \forall t_1, t_2 \in r$ cu $t_1[A_1 \dots A_n] = t_2[A_1 \dots A_n]$

$\Rightarrow t_1[B_1 \dots B_m] = t_2[B_1 \dots B_m]$ (I)

Din FD1, $Z \subseteq W \Rightarrow W \to Z \Rightarrow$ dc $t_1[C_1 \dots C_p] = t_2[C_1 \dots C_p]$

$\Rightarrow t_1[C_{i1} \dots C_{ik}] = t_2[C_{i1} \dots C_{ik}]$ (II)

Fie $t_1, t_2 \in r$ a.î. $t_1[XW] = t_2[XW] \Leftrightarrow t_1[A_1, A_2, \dots, A_n, C_1, C_2, \dots, C_p] = t_2[A_1, A_2, \dots A_n, C_1, C_2, C_p]$

$\Rightarrow \left\{\begin{array}{l} t_1[A_1, A_2, A_n] = t_2[A_1, \dots, A_n] \overset{(I)}{\Rightarrow} t_1[B_1 \dots B_m] = t_2[B_1 \dots B_m] \\ t_1[C_1, C_2, \dots C_p] = t_2[C_1, C_2, \dots, C_p] \overset{(II)}{\Rightarrow} t_1[C_{i1} \dots C_{ik}] = t_2[C_{i1} \dots C_{ik}] \end{array}\right\} \Rightarrow t_1[YZ] = t_2[YZ]$

$\Rightarrow$ în concluzie $\forall t_1, t_2 \in r$ cu $t_1[XW] = t_2[XW] \Rightarrow t_1[YZ] = t_2[YZ] \Rightarrow XW \to YZ$

FD3 (Tranzitivitate) Dc. r satisface $X \to Y$, $Y \to Z$, atunci r satisface $X \to Z$.

$X = (A_1, A_2, \dots, A_n)$

$Y = (B_1, B_2, \dots, B_m)$

$Z = (C_1, C_2, \dots, C_p)$

$\forall t_1, t_2 \in r$ cu $t_1[A_1, A_2, \dots, A_n] = t_2[A_1 \dots A_n] \Rightarrow t_1[B_1 \dots B_m] = t_2[B_1 \dots B_m] \Rightarrow$

$\forall t_1, t_2 \in r$ cu $t_1[B_1 \dots B_m] = t_2[B_1 \dots B_m] \Rightarrow t_1[C_1 \dots C_p] = t_2[C_1 \dots C_p]$

$\forall t_1, t_2 \in r$ cu $t_1[A_1 \dots A_n] = t_2[A_1 \dots A_n] \Rightarrow t_1[C_1 \dots C_p] = t_2[C_1 \dots C_p] \Rightarrow X \to Z$

FD4 (Pseudotranzitivitate) Dc. se satisface $X \to Y$ și $YW \to Z$, atunci se satisface $XW \to Z$.

1. $X \to Y$ (ip)
2. $YW \to Z$ (ip)
3. $W \subseteq W$ (th. mulț.)
4. $XW \to YW$ (1, 3, FD2f)
5. $XW \to Z$ (4, 2, FD3f)

FD5 (Uniune) Dc. se satisface $X \to Y$ și $X \to Z$, atunci se satisface $X \to YZ$.

1. $X \to Y$ (ip)
2. $X \to Z$ (ip)
3. $Y \subseteq Y$ (th. mulț.)
4. $XY \to YZ$ (2, 3, FD2f)
5. $X \subseteq X$ (th. mulț.)
6. $\underbrace{XX}_{X} \to XY$ (1, 5, FD2f)
7. $X \to YZ$ (6, 4, FD3f)

FD6 (Descompunere) Dc. se satisface $X \to YZ$, atunci se satisface $X \to Y$ și $X \to Z$.

1. $X \to YZ$ (ip)
2. $Y \subseteq YZ$ (th. mulț.)
3. $YZ \to Y$ (2, FD1f)
4. $X \to Y$ (1, 3, FD3f)

1. $X \to YZ$ (ip)
2. $Z \subseteq YZ$
3. $YZ \to Z$ (2, FD1f)
4. $X \to Z$ (1, 3, FD3f)