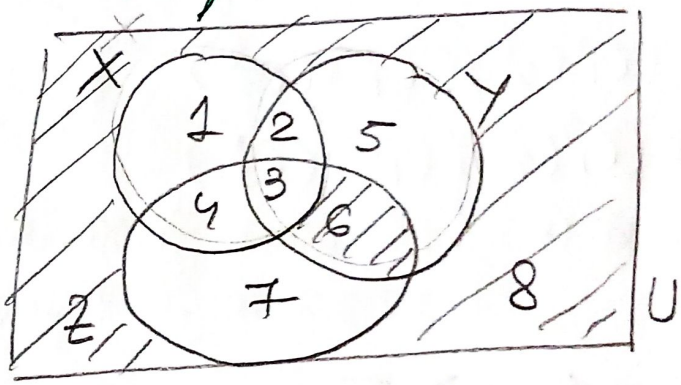
- **Proprietăți** ale dependențelor multivalorate

MVD0 (Complementaritate) Fie $X, Y, Z \subseteq U$ a.î. $XYZ = U$ și $Y \cap Z \subseteq X$. Dacă r satisface $X \twoheadrightarrow Y$, atunci r satisface $X \twoheadrightarrow Z$.



$Z = C_3 \cup C_4 \cup \underbrace{C_6}_{\emptyset} \cup C_7$

Dem. $t = (C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8)$

$C_8 = \emptyset$ $(XYZ = U)$

$C_6 = \emptyset$ $(Y \cap Z \subseteq X)$

$X \twoheadrightarrow Y \Rightarrow \forall t_1, t_2 \in r$ cu $t_1[X] = t_2[X] \Rightarrow \exists t_3, t_4$ a.î.:

$$\begin{cases} t_3[X] = t_1[X] & t_3[Y] = t_1[Y] & t_3[R_{XY}] = t_2[R_{XY}] \\ t_4[X] = t_2[X] & t_4[Y] = t_2[Y] & t_4[R_{XY}] = t_1[R_{XY}] \end{cases} \quad (R_{XY} = U \setminus \{X \cup Y\})$$

$R_{XY} = C_7 \cup C_8 = C_7$

Fie $t_1 = (C_1', C_2', \ldots, C_8')$

$t_1 = (C_1' C_2' C_3' C_4')(C_2' C_3' C_5')(C_7')$

proiecția relației pe X | pe Y | pe Z

$t_2 = (C_1^2 C_2^2 C_3^2 C_4^2)(C_2^2 C_3^2 C_5^2)(C_7^2)$

a.î. $t_1[X] = t_2[X] \Leftrightarrow (C_1' C_2' C_3' C_4') = (C_1^2 C_2^2 C_3^2 C_4^2)$

$\exists\, t_3, t_4$ a.î. $\begin{cases} t_3 = (C_1' C_2' C_3' C_4')(C_2' C_3' C_5')(C_7^2) & (*) \\ t_4 = (C_1^2 C_2^2 C_3^2 C_4^2)(C_2^2 C_3^2 C_5^2)(C_7') & (**) \\ \text{unde } C' \end{cases}$

$X \twoheadrightarrow Z$ înseamnă că $\forall t_1, t_2$ cu $t_1[X] = t_2[X] \Rightarrow \exists t_3', t_4'$ a.î.

$\begin{cases} t_3'[X] = t_1[X] & t_3'[Z] = t_1[Z] & t_3'[R \setminus XZ] = t_2[R \setminus XZ] \\ t_4'[X] = t_2[X] & t_4'[Z] = t_2[Z] & t_4'[R \setminus XZ] = t_1[R \setminus XZ] \end{cases}$

Plecăm de la $X \twoheadrightarrow Y$. Redenumim coloanele din t_1, t_2:

$\begin{cases} t_1 = (C_1' C_2' C_3' C_4')(C_3' C_4' C_7')(C_5') \\ t_2 = (C_1^2 C_2^2 C_3^2 C_4^2)(C_3^2 C_4^2 C_7^2)(C_5^2) \end{cases}$

Dacă $t_1[X] = t_2[X] \Rightarrow C_1' = C_1^2,\ C_2' = C_2^2,\ C_3' = C_3^2,\ C_4' = C_4^2$ (***)

$\exists t_3'', t_4''$ a.î.

$t_4'' = (C_1' C_2' C_3' C_4')(C_3' C_4' C_7^2)(C_5')$ (din (*), înlocuim forma lui t_3)

$t_3'' = (C_1^2 C_2^2 C_3^2 C_4^2)(C_3^2 C_4^2 C_7')(C_5^2)$ (din (**), înlocuim forma lui t_4)

$\Rightarrow$ Din (***), $t_3' = t_3'' = (C_1' C_2' C_3' C_4')(C_3' C_4' C_7')(C_5^2)$

$t_4' = t_4'' = (C_1^2 C_2^2 C_3^2 C_4^2)(C_3^2 C_4^2 C_7^2)(C_5')$

MVD1 (Reflexivitate) Dacă $Y \subseteq X$, atunci orice relație r satisface $X \twoheadrightarrow Y$.

Fie $t_1 = (C_1, C_2, C_3) = (\underbrace{C_1' C_2'}_{X})(\underbrace{C_1'}_{Y})(\underbrace{C_3'}_{R \setminus XY})$

$t_2 = (C_1^2, C_2^2, C_3^2) = (C_1^2 C_2^2)(C_1^2)(C_3^2)$

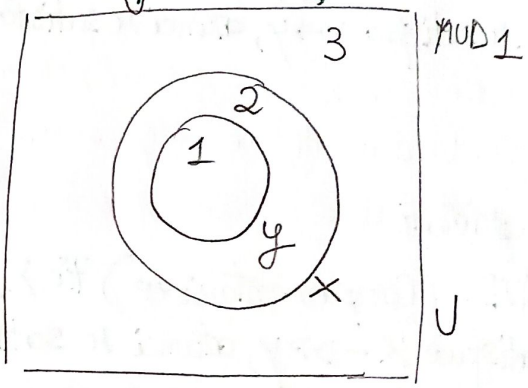
a.î. $t_1[X] = t_2[X]$ (adică $C_1' = C_1^2$ și $C_2' = C_2^2$) (*)

Avem de demonstrat că $\exists t_3, t_4 \in r$ a.î.

$t_3 = (C_1' C_2')(C_1')(C_3^2)$

$t_4 = (C_1^2 C_2^2)(C_1^2)(C_3')$

Datorită (*), $\begin{cases} t_1 \text{ poate fi rescris ca } t_4 = (C_1^2 C_2^2)(C_1^2)(C_3') \\ t_2 \text{ poate fi rescris ca } t_3 = (C_1' C_2')(C_1')(C_3^2) \end{cases}$



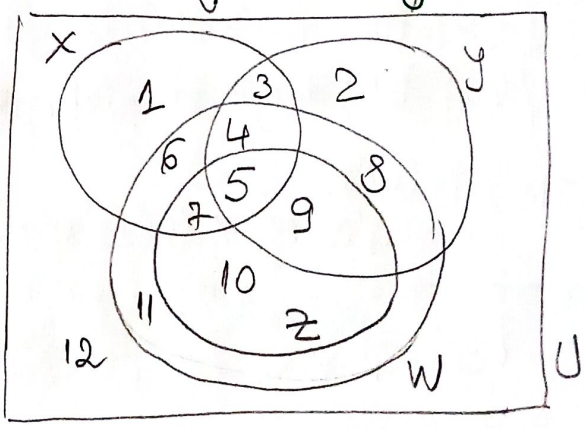
MVD2 (Extensie) Fie $Z \subseteq W$ și r satisface $X \twoheadrightarrow Y$. Atunci r satisface $XW \twoheadrightarrow YZ$.

Fie $t_1, t_2 \in r$ cu $t_1[X] = t_2[X]$. Dacă r satisface $X \twoheadrightarrow Y$:

$t_1 = (\underbrace{C_1' C_3'}_{X} - C_7')(\underbrace{C_2' - C_5' C_8' C_9'}_{Y})(\underbrace{C_{10}' - C_{12}'}_{R \setminus XY})$

$t_2 = (C_1^2 C_3^2 - C_7^2)(C_2^2 - C_5^2 C_8^2 C_9^2)(C_{10}^2 - C_{12}^2)$

$\exists t_3, t_4$ a.î. $\begin{cases} t_3 = (C_1' C_3' - C_7')(C_2' - C_5' C_8' C_9')(C_{10}^2 - C_{12}^2) \\ t_4 = (C_1^2 C_3^2 - C_7^2)(C_2^2 - C_5^2 C_8^2 C_9^2)(C_{10}' - C_{12}') \end{cases}$



Dem că $XW \twoheadrightarrow YZ$

Fie t_1', t_2' a.î. $t_1'[XW] = t_2'[XW] \Leftrightarrow \boxed{C_1' = C_1^2, C_3' = C_3^2, \ldots, C_{11}' = C_{11}^2}$ (*)

$\begin{cases} t_1' = (C_1' C_3' - C_{11}')(C_2' - C_5' C_7' - C_{10}')(C_{12}') \\ t_2' = (C_1^2 C_3^2 - C_{11}^2)(C_2^2 - C_5^2 C_7^2 - C_{10}^2)(C_{12}^2) \end{cases}$

Trebuie să dem. că $\exists \begin{cases} t_3' = (C_1' C_3' - C_{11}')(C_2' - C_5' - C_7' - C_{10}')(C_{12}^2) \\ t_4' = (C_1^2 C_3^2 - C_{11}^2)(C_2^2 - C_5^2 - C_7^2 - C_{10}^2)(C_{12}') \end{cases}$

Putem rescrie t_3 ca: $t_3 = (C_1' C_3' - C_9' C_{10}^2 C_{11}^2)(C_2' - C_5' C_7' - C_8' C_{10}^2)(C_{12}^2)$

Din (*) $\Rightarrow t_3' = t_3$

Putem rescrie t_4 ca: $t_4 = (C_1^2 C_3^2 - C_9^2 C_{10}' C_{11}')(C_2^2 - C_5^2 C_7^2 - C_9^2 C_{10}')(C_{12}')$

Din (x) $\Rightarrow t_4' = t_4$

MVD3 (Tranzitivitate) Dc. x satisface $X \twoheadrightarrow Y$ și $Y \twoheadrightarrow Z$ at. x satisface $X \twoheadrightarrow Z-Y$.

$X \twoheadrightarrow Y$. Fie t_1, t_2 ai $t_1[X] = t_2[X]$

$t_1 = (C_1' C_2' C_3' C_4')(C_2' C_3' C_5' C_6')(C_7' C_8')$, $\exists t_3, t_4$ ai. $t_3 = (C_1' C_2' C_3' C_4')(C_2' C_3' C_5' C_6')(C_7^2 C_8^2)$

$t_2 = (C_1^2 C_2^2 C_3^2 C_4^2)(C_2^2 C_3^2 C_5^2 C_6^2)(C_7^2 C_8^2)$ (*) $t_4 = (C_1^2 C_2^2 C_3^2 C_4^2)(C_2^2 C_3^2 C_5^2 C_6^2)(C_7' C_8')$

$Y \twoheadrightarrow Z$ Fie t_1', t_2' ai. $t_1'[Y] = t_2'[Y]$

$t_1' = (C_2^1 C_3^1 C_5^1 C_6^1)(C_3^1 C_4^1 C_6^1 C_7^1)(C_1^1 C_8^1)$, $\exists t_3', t_4'$ ai $t_3' = (C_2^1 C_3^1 C_5^1 C_6^1)(C_3^1 C_4^1 C_6^1 C_7^1)(C_1^2 C_8^2)$

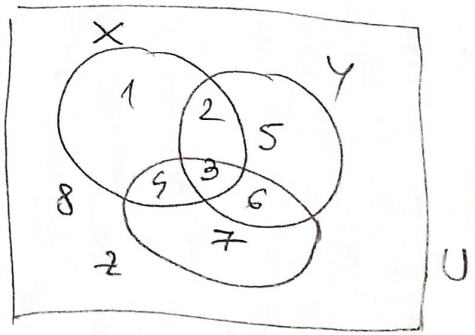
$t_2' = (C_2^2 C_3^2 C_5^2 C_6^2)(C_3^2 C_4^2 C_6^2 C_7^2)(C_1^2 C_8^2)$ (**) $t_4' = (C_2^2 C_3^2 C_5^2 C_6^2)(C_3^2 C_4^2 C_6^2 C_7^2)(C_1^1 C_8^1)$

Dem că $X \twoheadrightarrow Z-Y$, Fie t_1'', t_2'' ai $t_1''[X] = t_2''[X]$

$t_1'' = (C_1' C_2' C_3' C_4')(C_4' C_7')(C_5' C_6^1 C_8')$

$t_2'' = (C_1^2 C_2^2 C_3^2 C_4^2)(C_4^2 C_7^2)(C_5^2 C_6^2 C_8^2)$

Trebuie să dem că $\exists \begin{cases} t_3'' = (C_1' C_2' C_3' C_4')(C_4' C_7')(C_5^2 C_6^2 C_8^2) \\ t_4'' = (C_1^2 C_2^2 C_3^2 C_4^2)(C_4^2 C_7^2)(C_5' C_6' C_8') \end{cases}$



$t_3' = (C_1^2 C_2^1 C_3^1 C_4^1)(C_4' C_7')(C_5^2 C_6' C_8^2) = t_3''$

C_1^1) din (*) $C_5^2 C_6^2$ din (**)

$t_4' = (C_1' C_2^2 C_3^2 C_4^2)(C_4^2 C_7^2)(C_5^2 C_6^2 C_8') = t_4''$

C_1^2) din (*) $C_5' C_6'$ din (**)

MVD4 (Reuniune)