C3 1. Non-deterministic algorithms

- for some configurations there are more than one way to continue the execution
- consequently, for the same input the algorithm may have many executions with different results
- execution time: the time of the execution that leads to the correct result

`choose` x `from` S;
– returns an element from S, arbitrarily chosen
– execution time (uniform): $O(1)$

`choose` x `from` S `s.t.` B;
– returns an element from S that satisfies B
– equivalent to
`choose` x `from` S;
`if` $(\neg B(x))$ failure;
– execution time: $T(B)$

```
odd(x) {
  return x % 2 == 1;
}

L = emptyList;
for (i = 0; i < 8; i = i+2)
  L.pushBack(i);
choose x from L s.t. odd(x);

$ alki -a failure.alk -m

failure
s |-> < 0, 2, 4, 6 >
i |-> 8

Note that the executed algorithm is nondeterministic.
```

Conclusion: an execution of an nondeterministic algorithm may fail!

- there a two main steps:
  - first "guesses" a certain structure S
  - then checks if S satisfies the property requested by the question
  - if yes the the execution finishes with `success`, otherwise it finishes with `failure`;
- Extending the language:

  `success;` – signals the successful termination of an execution
  `failure;` – signals the termination of a failing execution

SAT

@instance A set of n propositional variables and a propositional formula F in conjunctive normal form.
@question Is F satisfiable?

```
// guess
for (i = 0; i < n; ++i) {
  choose z in {false, true};
  x[i] = z;
}
// check
if (f(x)) success;  // f((x) describes F
else failure;
```

2. Randomized(Probabilistic) algorithms

Works with random variables

E.g., random variable for the sum of values on two dice

x_i	2	3	4	5	...
p_i	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	...

The instruction

```
uniform x from S;
```

assigns to x a value uniformly chosen from the iterable data type S.
Execution time: $O(1)$
uniform-test.alk:

```
uniform x from {0..4};
print(x);
```

Consider only discrete random variable X, whose values are real numbers $x_1, x_2, \ldots$.

$p_i = Pr(X = x_i)$ - probability as X to have the value x_i

Expected Value of X: $E(X) = \sum_i x_i \cdot p_i$

Properties:
$E(X + Y) = E(X) + E(Y)$
$E(X \cdot Y) = E(X) \cdot E(Y)$
(X și Y independent)